Surface growth with spatially correlated noise

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The $(2+1)$-dimensional surface growth with spatial long-range correlations is studied using the Kadar-Parisi-Zhang equation. The growth exponent is found to increase with the parameter ρ, which characterizes the noise correlations, especially for $\rho>0.5$. [S1063-651X(97)02501-4]

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Nonequilibrium surface growth processes often exhibit a phenomenon called kinetic roughening, where the surface develops a self-affine morphology [1]. Much attention has been given to a special class of models (ballistic deposition, Eden, or polynucleation growth), which are described by the Kardar-Parisi-Zhang (KPZ) equation [2]

$$\partial_t h(\vec{x},t)=\nu\nabla^2 h(\vec{x},t)+\frac{\lambda}{2}[\nabla h(\vec{x},t)]^2+\eta(\vec{x},t). \qquad (1)$$

Here $h(\vec{x},t)$ is the local height of the surface above a d-dimensional substrate in a $(d+1)$-dimensional space, λ characterizes the tilt dependence of the growth velocity, ν is an effective surface tension, and $\eta(\vec{x},t)$ is the noise.

Solutions of Eq. (1) show the scaling behavior. The simplest quantity to investigate is the surface width $W=\langle[\overline{h^2}-\overline{h}^2]^{1/2}\rangle$, where the overbar and angular brackets denote spatial and noise averages, respectively. For a system of size L,W is suggested to have the scaling form [3]

$$W(L,t)\sim\begin{cases} t^{\beta} & \text{if } t\ll L^z \\ L^{\alpha} & \text{if } t\gg L^z, \end{cases} \qquad (2)$$

where β, α, and z are the growth, roughness, and dynamic exponent of the interface, respectively.

The case when the noise is uncorrelated has been well studied. For the $(1+1)$ dimension one can obtain exact results $\beta=1/3$ and $\alpha=1/2$ by mapping Eq. (1) into the Burgers equation [2]. For $d>1$ the exact results are lacking and the critical exponents have been evaluated numerically [1].

Recently, Medina *et al.*, [4] considered the spatially correlated noise with the correlator

$$\langle\eta(\vec{x},t)\eta(\vec{x'},t')\rangle=2D|\vec{x}-\vec{x}'|^{2\rho-d}\delta(t-t'), \qquad (3)$$

where $0\leq\rho\leq d/2$. The existence of long-range correlated noise has been suggested as a possible mechanism for anomalous exponents observed in experiments [5] as well as in simulations [6] on interface growth in porous media. By a dynamical renormalization group it is found [4] that in $1+1$ dimensions for small ρ the critical exponents are the same as for uncorrelated noise. For ρ above a certain critical value the exponents become dependent on ρ. It should be noted that different theoretical approaches [4,7,8] lead to different dependences of the critical exponents on ρ in $1+1$ dimensions. To check theoretical predictions numerical simulations have been carried out for the KPZ equation [9], the ballistic deposition [9–11], the directed polymer [9], and the solid-on-solid model [11]. There are some disagreements among the simulation results. For example, the results of Amar, Lam, and Family [11] obtained for the ballistic deposition and restricted solid-on-solid models agree with the prediction of Medina *et al.* [4], but are in conflict with the prediction of Zhang [7]. Numerical studies [9] of the effect of the long-range spatially correlated noise on the surface growth described by the KPZ equation and on the related directed polymer problem, on the other hand, give good agreement with the prediction of Hentschel and Family [8]. The surface growth with temporally correlated noise has been studied also in $(1+1)$ dimensions [4,12].

It should be noted that the $(2+1)$-dimensional KPZ equation with a special type of noise correlations, namely, $\langle\eta(\vec{x},t)\eta(\vec{x}',t')\rangle\sim[(\vec{x}-\vec{x}')^2+(t-t')^2]^{-1/2}$, has been studied in Ref. [13] to analyze roughness of vortex lines in the random gauge XY model. The $(2+1)$-dimensional KPZ equation with correlated noise given by Eq. (3), however, has not been investigated yet. It has been shown only that the critical exponents cannot be obtained by the dynamical renormalization-group approach for this case because fixed points do not exist in the one-loop approximation [4]. In a related paper, Meakin and Jullien [10,14] introduced a hopping model of ballistic deposition, in which particles were deposited on the growing surface following a Levy flight distribution. In this case, the distance measured between sites of subsequent growth, as measured along the interface, is chosen to be equal to $x=r^{-1/f}$, where r is a random number between 0 and 1. The authors have claimed that this model should be described by the KPZ equation with ρ of Eq. (3) being equal to $\frac{1}{2}f$. The results obtained for the exponents α and β roughly agree with the prediction of Medina *et al.* for $1+1$ dimensions [10]. A weak dependence of these exponents on f was found in the $(2+1)$-dimensional case [14].

It should be noted that in the simulations of Meakin and Jullien the link between the deposition process and the noise is rather unclear. As it was mentioned in Ref. [11], the Levy flight deposition rule used by these authors would not lead to the expected distribution $P(x)\sim x^{2\rho-1}$. Thus the scaling behavior in $(2+1)$-dimensional surface growth with spatially correlated noise remains, at the present time, unclear.

In this paper we study this problem solving the KPZ equation numerically. We find that for $\rho>0.5$ the critical exponents have a stronger dependence on ρ compared to those obtained in the Meakin-Jullien model [14].

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The spatial derivatives in Eq. (1) are discretized using standard forward-backward differences on a hypercubic grid with a lattice constant Δx. The integration of this equation is carried out by the Euler algorithm with time increments Δt. Denoting the grid points by $\vec{n}$ and the basic vectors characterizing the surface by $\vec{e}_1,\ldots,\vec{e}_d$ we arrive at the discretized equation [15]

$$\tilde{h}(\vec{n},\tilde{t}+\delta\tilde{t})=\tilde{h}(\vec{n},\tilde{t})+\frac{\Delta\tilde{t}}{\Delta\tilde{x}^2}\sum_{i=1}^{d}\{[\tilde{h}(\vec{n}+\vec{e}_i,\tilde{t})-2\tilde{h}(\vec{n},\tilde{t}) +\tilde{h}(\vec{n}-\vec{e}_i,\tilde{t})]+\tfrac{1}{8}[\tilde{h}(\vec{n}+\vec{e}_i,\tilde{t}) -\tilde{h}(\vec{n}-\vec{e}_i,\tilde{t})]^2\}+\sqrt{3\Delta\tilde{t}}\,\eta(\vec{n},\tilde{t}). \quad (4)$$

Here one uses dimensionless quantities $\tilde{h}=h/h_0$, $\tilde{x}=x/x_0$, and $\tilde{t}=t/t_0$, where the natural units are given by

$$h_0=\frac{\nu}{\lambda},\quad t_0=\frac{\nu^2}{\sigma^2\lambda^2},\quad x_0=\sqrt{\frac{\nu^3}{\sigma^2\lambda^2}},\quad \sigma^2=2D/\Delta x^d. \quad (5)$$

In the Fourier space the renormalized noise $\eta(\vec{n},\tilde{t})$ has the correlation

$$\langle\eta(\vec{k},\omega)\,\eta(\vec{k}',\omega')\rangle=k^{-2\rho}\delta(\vec{k}+\vec{k}')\,\delta(\omega+\omega'). \quad (6)$$

Since the spatially correlated noise does not break the invariance of Eq. (2) with respect to a tilting of the surface [3] the exponents α, z, and β are related by the scaling laws

$$\alpha+z=2,\quad z=\alpha/\beta. \quad (7)$$

Thus, to obtain a full set of the critical exponents one has to calculate one of them. We will determine β as a function of ρ.

To create the spatially correlated noise we follow Peng *et al* [9]. We first generate a standard white (or Gaussian uncorrelated) noise $\eta_0(\vec{n},\tilde{t})$ and then carry out the Fourier transformation for spatial variables to obtain $\eta_0(\vec{q},\tilde{t})$. We define

$$\eta(\vec{q},\tilde{t})=|\vec{q}|^{-\rho}\eta_0(\vec{q},\tilde{t}). \quad (8)$$

The noise $\eta(\vec{n},\tilde{t})$ is obtained by Fourier transforming $\eta(\vec{q},\tilde{t})$ back into the space domain. The fast Fourier transformation algorithm [16] has been implemented.

In our simulations we consider the case of 2+1 dimensions and choose Δx, ν, and σ to be the same as in Ref. [15], namely, $\Delta x=1$, $\nu=0.5$, and $\sigma=0.1$. To be sure that we are in a strong-coupling regime we chose $\lambda=\sqrt{600}$. The time increment Δt should be small enough to ensure the stability of algorithm. Simple von Neumann stability analysis [15] for the corresponding linear equation ($D=0$, $\lambda=0$) shows that Δt must satisfy the condition $\Delta\tilde{t}<(\Delta\tilde{x})^2/2$. We found that Δt should be much smaller than the upper bound given by this criterion and its choice strongly depends on ρ. The larger the value of ρ, the smaller Δt should be taken. Depending on ρ, we choose Δt between 0.02 and 0.000 25. The number of samples used for noise averaging is taken between 50 and 100. For each value of ρ we calculate the noise-noise correlation function and compare it to the expected $x^{2\rho-2}$ behavior. For $\rho<0.5$ the correlation function of noise behaves as expected, but for larger ρ the effective decay exponent for the generated noise (ρ') was found somewhat smaller. For the largest $\rho=1$ we obtained $\rho'\approx0.82$. In what follows ρ should be understood as the effective exponent ρ'.

Figure 1 shows the time dependence of the width W for the system size $L=256$ for various values of ρ. For $\rho=0$ we reproduce the result $\beta=0.241\pm0.004$ of Moser, Kertesz, and Wolf [15]. The dependence of β on ρ is shown in Fig. 2. For $\rho<0.35$ the long-range correlations are probably still irrelevant and the values of β are approximately equal to those

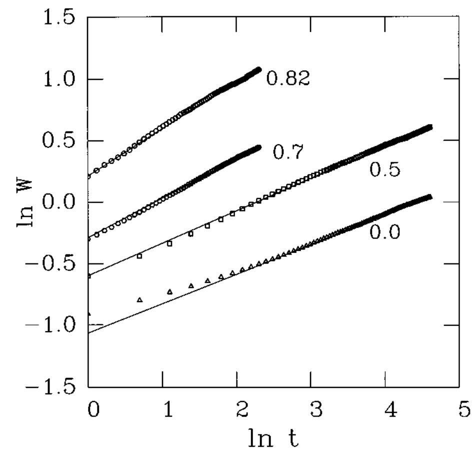

FIG. 1. log-log plot of the width W vs t for different values of ρ. Here $L=256$, $\Delta x=1$, $\nu=0.5$, $\sigma=0.1$, and $\lambda=\sqrt{600}$. Open triangles, squares, hexagons, and cirles correspond to $\rho=0$, 0.5, 0.7, and 0.82, respectively. For $\rho\leqslant0.5$ we take $\Delta t=0.02$ and the results are averaged over 100 samples. For $\rho=0$ the best fit gives $\beta=0.241\pm0.004$, which agrees with the value reported by Moser, Kertesz, and Wolf [15]. For $\rho=0.7$ and 0.82 we chose $\Delta t=0.0005$ and 0.000 25, respectively. The average is taken over 50 runs. The error bars are smaller than the symbols.

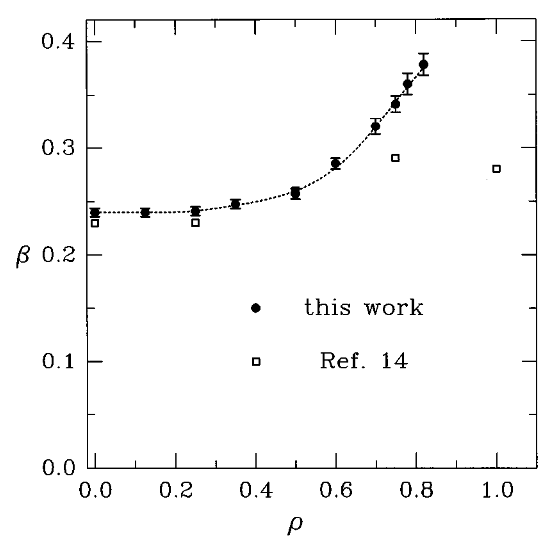

FIG. 2. Dependence of β on ρ. The error bars are due to the fitting procedure. The results from Ref. [14] (open squares) are also shown for comparison.

obtained without the correlated noise. The increase of β becomes more significant for $\rho \geqslant 0.5$. For $\rho > 0.5$ our values of β are higher than those obtained by Meakin and Julien for the hopping model of ballistic deposition [14]. For the largest $\rho = 0.82$ we obtain $\beta = 0.38 \pm 0.01$.

To summarize, we have obtained a nonuniversal growth exponent β for the surface growth in $2+1$ dimensions with spatially correlated noise by integrating the KPZ equation. Other critical exponents may be obtained from the scaling laws. Our results suggest that the model of ballistic deposition with the Levy flight distribution of particles [14] is not equivalent to the KPZ equation with long-range spatially correlated noise in $2+1$ dimensions. It would be interesting to study the effect of temporal noise on the growth processes in $(2+1)$-dimensional systems.

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